Rail-Structure Interaction
in accordance with UIC774-3

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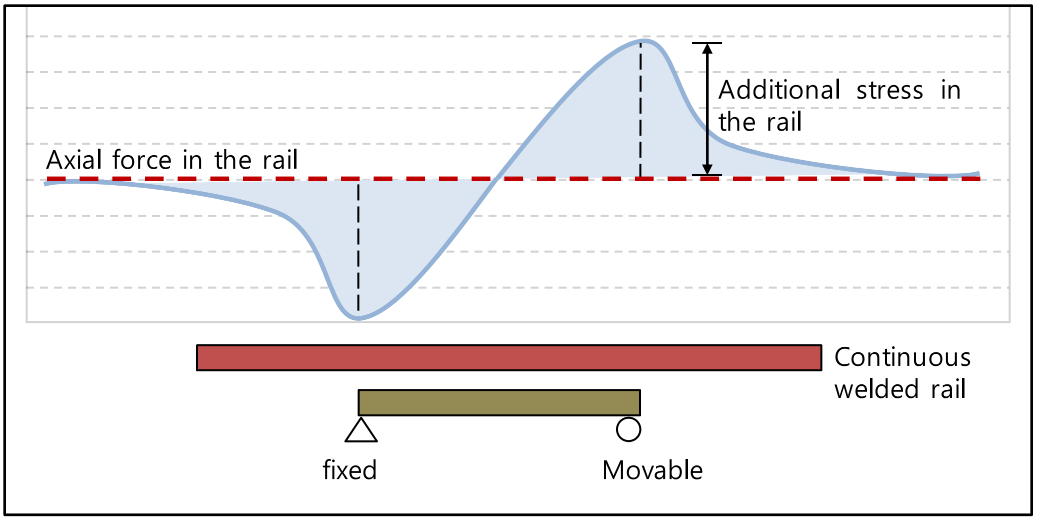
00. Contents

01. Overview

The continuous welded rail on the bridge is connected to the superstructure by the ballast and thus, the bridge and the track interact due to loading. When thermal loading is applied, expansion or contraction presents in the bridge and the rail. Now that the rail is supported by a fastener, crosstie, ballast, etc, expansion and contraction is restrained and accordingly, the axial force is accumulated in the rail. And because the load from the bridge deck is transferred to the rail through the ballast, the axial force occurs in the continuous welded rail. Also the accelerator or brake load of the train causes the axial force, and the varying bending moment of the deck due to the vertical load causes the axial force.



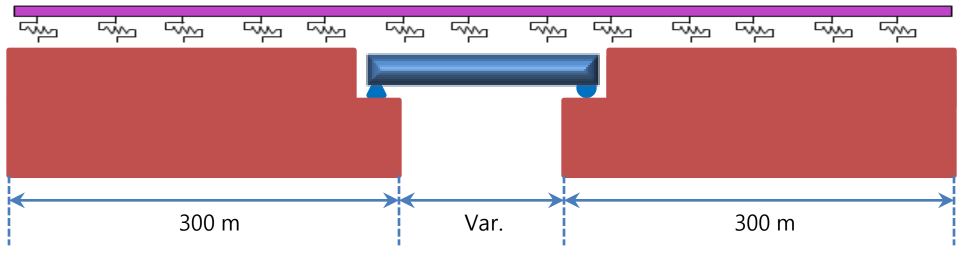
| Axial force in the continuous welded rail |

The above figure is the axial force diagram for the continuous welded rail on the bridge., The additional stresses in the rail are compared with the allowable additional stresses stated in the Code for thermal loading, accelerator/brake loading and vertical loading separately, to check the safety.

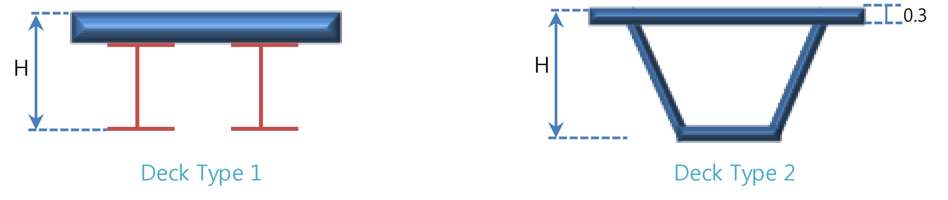
This technical paper is the verification document studying the axial forces in the continuous welded rail using the Multi-Linear Elastic Link. The testing has been performed according to UIC774-3.

2.1 Test Cases in the UIC774-3 Code

UIC774-3 states that testing should be carried out according to Appendix 1.7.1 and the error should be within 10%. However, as long as it is on the safety side, an error of 20% is allowed. Many test cases are outlined in the Code, but only two test cases called E1-3 and E4-6 are studied in this paper.



| Test model for a short span bridge described in UIC774-3 |



| Deck types used for the tests in UIC774-3 |

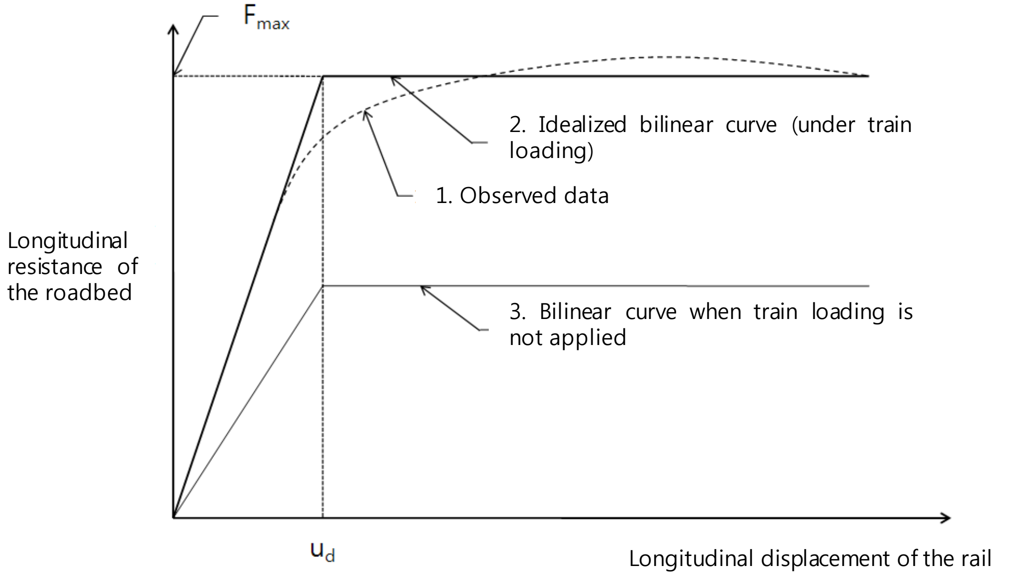
The deck types used for testing are as above and the properties are listed in the table below. E is modulus of elasticity, I is moment of inertia, H is height and S is cross sectional area. For simplicity, the neutral axis of the rail is assumed to be on the top of the reinforced concrete bridge deck. The properties that are not listed below are assumed properly.

Case No.	Deck Type	Span (m)	Direction of train load	K Long 🎧 (KN/m)	E (KN/m^2)	I (m^4)	H (m)	S (m^2)
E1-3	1	60	1(left→right)	600000	2.1E^8	2.59	6.0	0.74
E4-6	1	60	2(right→left)	600000	2.1E^8	2.59	6.0	0.74

🎧 K Long: the modulus of elasticity in the longitudinal direction for the bridge bearing. In the test model, Point Spring stiffness is entered.

2.2 Analysis Methods and Applications

Under thermal loading, the roadbed is characterized by the Multi-Linear Elastic Link, which represents the resistance in the unloaded state. Under rail loading (accelerator/brake loading, vertical loading), the loaded roadbed is characterized by the Multi-Linear Elastic Link, which represents the resistance in the loaded state.



| Longitudinal resistance of the roadbed |

According to the UIC Code, the analysis methods described below can be used for the track-bridge interaction analysis using software. Depending on the software, either of the methods can be used. But it should be noted that there can be an error between the two analyses.

1) Separate analyses for temperature variation, accelerator/brake load and vertical load

Two or more nonlinear models having each load alone are analyzed and the results are combined. This is the simplest way of analyzing the rail-bridge interaction as it assumes that the superposition of the analysis results is allowed, which is normally not allowed in the nonlinear analysis. According to the UIC774-3 Code, generally 20%~30% larger stress can occur in the rail.

2) Staged analysis for temperature variation, accelerator/brake load and vertical load

Construction Stage Analysis function of *midas CIVIL* is used. Before the train load is applied, the temperature load affects the structure. This initial displacement is reflected in the analysis in a stage when the train load is applied.

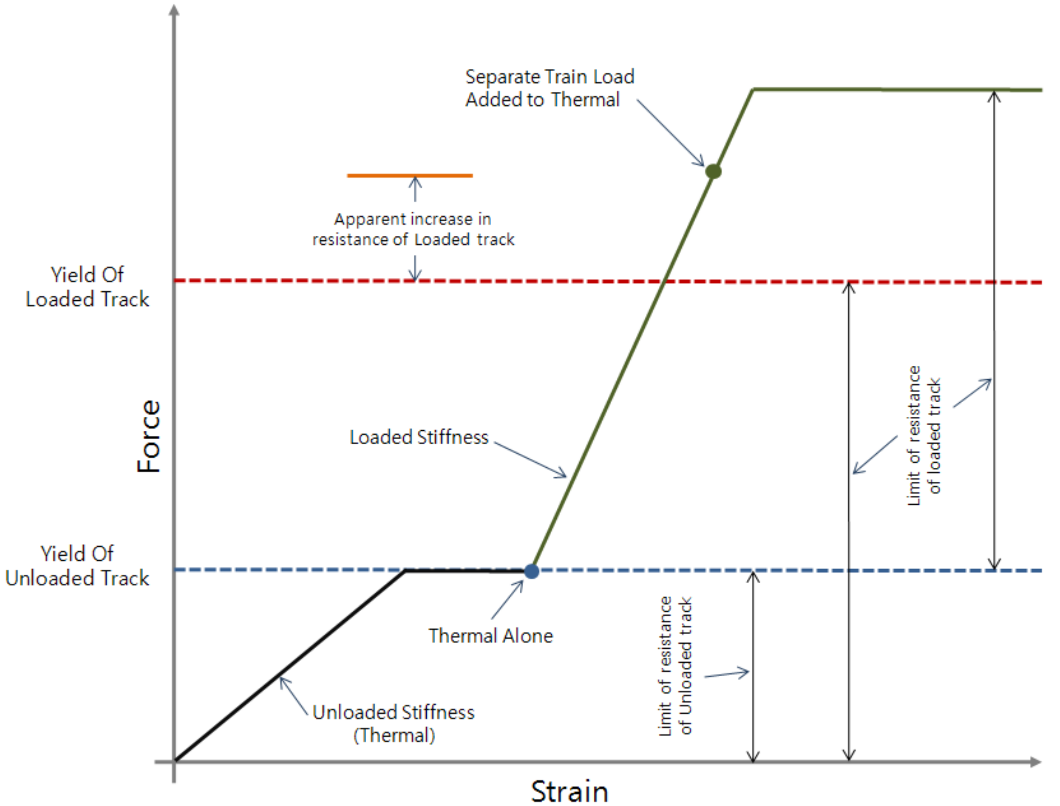
If a comparison is made between the two analyses, in general, the staged analysis reduces the compressive stress in the rail underneath the train as the yielding zone is larger in the staged analysis than the separate analysis.

Therefore, the separate analysis overestimates the axial force because the resistance of the ballast at which train loading is applied is estimated to be the resistance under train loading plus the resistance under thermal loading, instead of the resistance under train loading alone.

2.3 Description of each Analysis Method

2.3.1 Separate Analyses for Thermal Load, Accelerator/Brake Load and Vertical Load

As the effects of temperature are not known, the initial stress and strain of the structure prior to train loading is assumed to be zero. In other words, two or more nonlinear models having each load exclusively are analyzed and their results are combined. This is the simplest way of analyzing the rail-bridge interaction and it assumes that the superposition of results is allowed, which is normally not allowed for the nonlinear analysis.



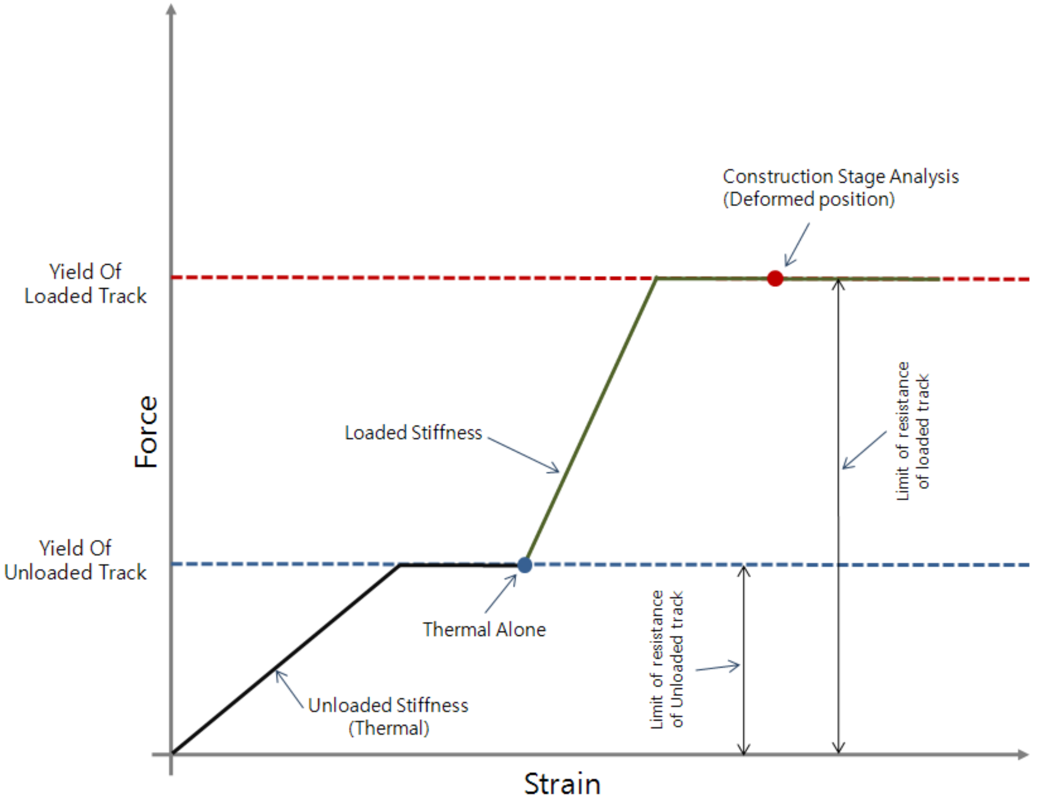
When the thermal load is applied separately, initially the resistance of the ballast to the horizontal direction follows the "Unloaded Stiffness" curve up to "Limit of resistance of unloaded track". That is, it follows the "Unloaded Stiffness" curve until it reaches "Thermal Alone".

In addition to this, if the train load is applied separately, the horizontal resistance starts from the "Thermal Alone" point and follows the "Loaded Stiffness" curve until it reaches "Separate Train Load Added to Thermal". The train load analysis uses "Loaded Stiffness", but the final result is obtained by combining the train load analysis result with the thermal load analysis result.

As a result, the ballast resists the extra load "Apparent increase in resistance of loaded track" besides the yield load under the "Loaded" state. Thus, the stress in the rail is overestimated.

2.3.2 Staged Analysis for Thermal Load, Accelerator/Brake Load and Vertical Load

Before the train load is applied, temperature has an effect on the structure. The initial state of strain is taken into account during train loading. In other words, when the relative displacement of the rail to the bridge has already occurred due to the temperature load, if the train load is applied to a certain location, the behavior of the roadbed at which the train is located switches from the "Unloaded Bi-Linear Curve" to the "Loaded Bi-Linear Curve" and the displacements and the internal forces in the rail and the structure due to the temperature load are maintained.



Like the separate analysis, in the thermal loading stage, the resistance reaches up to "Thermal Alone".

The biggest difference of the staged analysis compared to the separate analysis is explained below. The train load is applied while the displacements and the internal forces due to the thermal load are maintained. The resistance switches to the "Loaded Stiffness" curve over the roadbed where the train load is applied. The horizontal resistance follows the "Loaded Stiffness" curve from the "Thermal Alone" position, but the limit is from the origin to "Limit of resistance of loaded track", not from "Thermal Alone". The limit is indicated by "Construction Stage Analysis (Deformed position)".

This paper compares the separate analysis to the staged analysis for the test models in the UIC Code to demonstrate the reliability of *midas CIVIL* modeling methods and analysis results.

2.4 Thermal Load

The axial force occurs in the continuous welded rail as the expansion or contraction force due to temperature variation is accumulated in the rail. Thus, the temperature variation is the key parameter to the axial force in the continuous welded rail. The following table shows the temperature variations stipulated in each country code.

Code	Rail	Concrete Bridge	Steel Bridge
Continuous welded rail code (Korea)	△T=±50 ℃	△T=±25 ℃	Normal climate:△T=±35 ℃ Cold climate:△T=±45 ℃
UIC Code	△T=±50 ℃	△T=±35 ℃ Max temp var between the deck and the rail: ±20 ℃	△T=±35 ℃
Railway design code (Korea)	Avg temp 20 ~25 ℃ (± 3℃) Max rail temp 60 ℃ Min rail temp -20 ℃	±15 ℃	△T=±35 ℃ (normal dimate) (temp var -20~+50 ℃) △T=±45 ℃ (cold climate) (temp var -30~+50 ℃)
Shinkansen (Japan)	△T=±40 ℃		

| Temperature variation for the continuous welded rail analysis |

In this paper, the temperature variation specified in the UIC Code is applied to the rail and the concrete bridge.

· Rail : T=+50 ℃

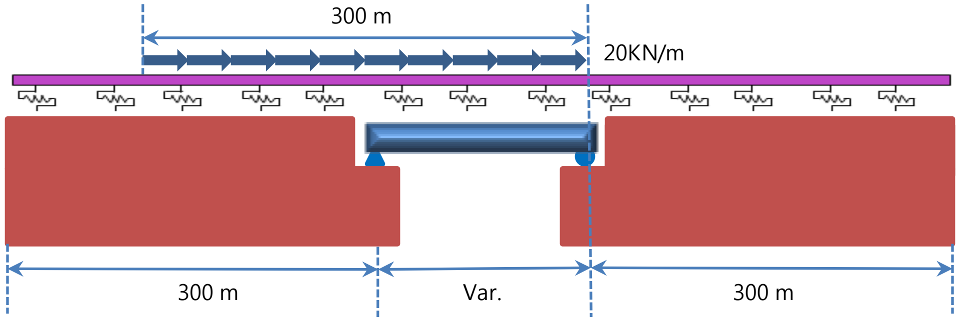
· Bridge : T=+35 ℃

2.5 Accelerator/Brake Load

Code	Accelerator Load	Brake Load
UIC Code	33kN/m below 1000kN	20kN/m (below 6000kN in total)
Continuous welded rail code (Korea)	33kN/m below 34m below 1000kN	20kN/m 400m (below 8000kN in total)
Railway design code (Korea)	33kN/m × L(m) ≤ 1000kN	20kN/m × L(m) ≤ 6000kN

| Accelerator/Brake load for the continuous welded rail analysis |

In this paper, the brake load is applied to the train length (300m) as shown in the figure below as presented in the UIC Code.

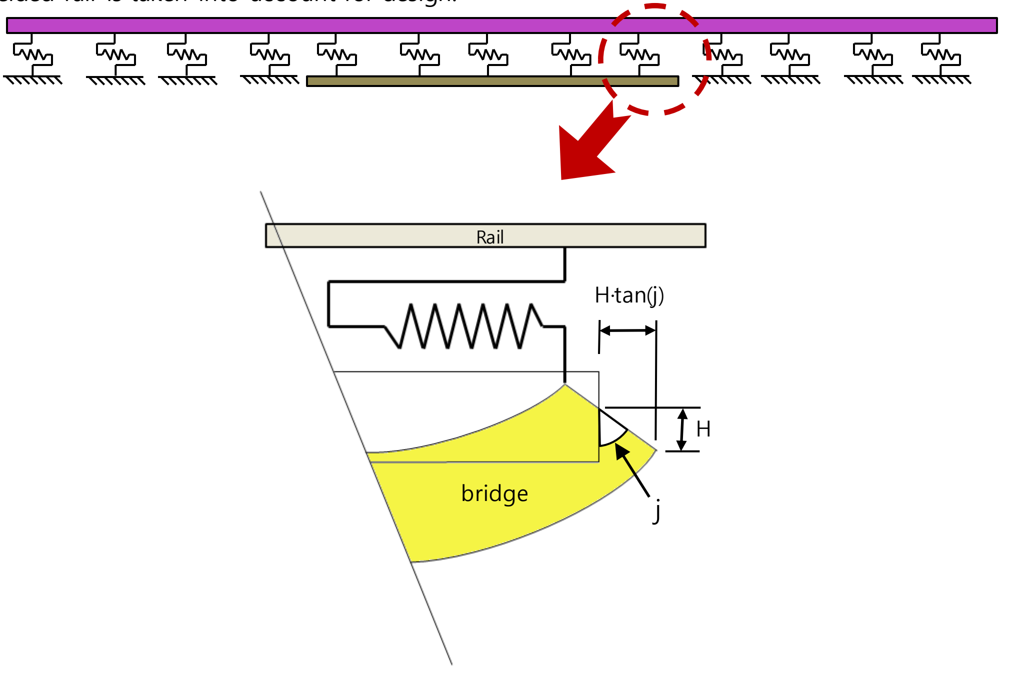


| Brake load for the continuous welded rail analysis |

To represent the bending deformation of the bridge slab due to the vertical load applied to the rail, the Elastic Link (Dx stiffness) is added between the rail and the slab.

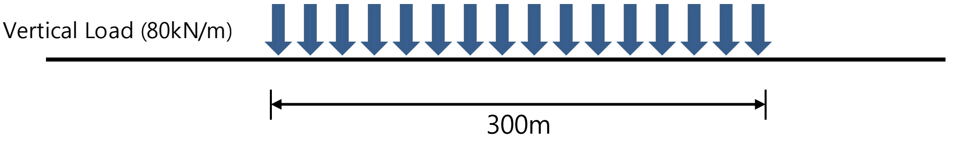
2.6 Vertical Load

It has been known that thermal load and accelerator/brake load are the important factors influencing the axial forces in the continuous welded rail on the bridge for design. However, according to the UIC Code, also the vertical train load affects the axial force in the track. To prevent the instability of the ballast on the roadbed due to the end angle of the bridge deck, it is required to check the longitudinal displacements at the top of the abutment end or at the top of the continuous deck end while the vertical train load is applied. In this example, the axial force due to the vertical train load (impact factor excluded) in the continuous welded rail is taken into account for design.



| The axial force occurs in the rail due to the deck bending |

As shown in the figure, when an external vertical load is applied to the bridge, as a bending occurs in the bridge deck and a longitudinal displacement occurs on top of the bridge deck, the axial force is induced in the rail. The interaction between the deck and the track is induced by the displacement through the medium of ballast. This displacement generates a large force in the track and the support, which induces the axial force in the track. To analyze this, the model is built such that the bending of the deck will cause the axial force in the track. The locations of the vertical train loads are identical to those of the brake loads.



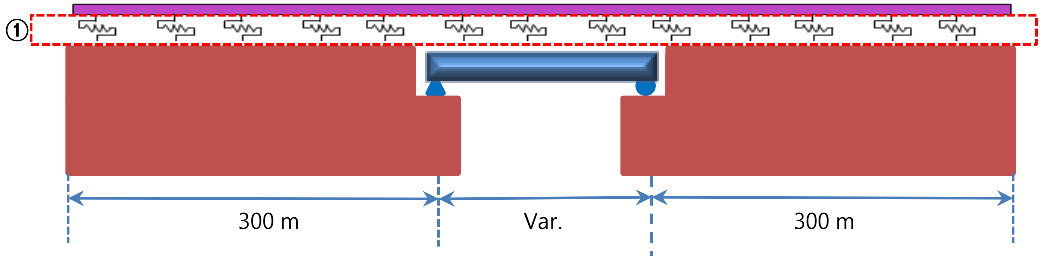
| The vertical load applied to the continuous welded rail |

3.1 Compose the Verification Model

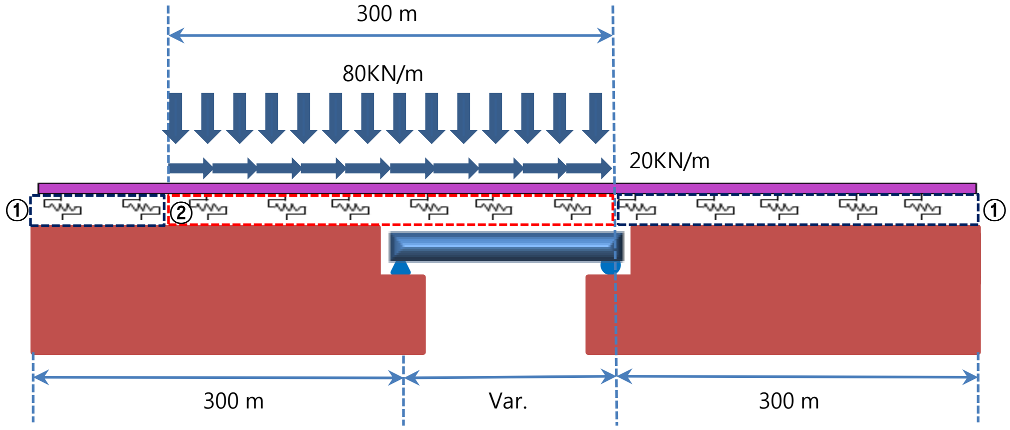
Compose the verification model for the staged analysis including Bi-Linear behavior.

Classification	Longitudinal Resistance Spring Constant of roadbed	Limit Displacement	Remark
When a load is not applied	20 kN/m	2 mm	①
When a load is applied	60 kN/m	2 mm	②

| Resistance spring constant of the roadbed in accordance with the UIC774-3 Code |



| When thermal load is applied |



| When train load is applied |

As shown in the figure above, Bi-Linear links to apply thermal load and train load are defined using the *midas CIVIL* Multi-Linear Elastic Link function. For the details on Multi-Linear Elastic Link, refer to Online Manual.

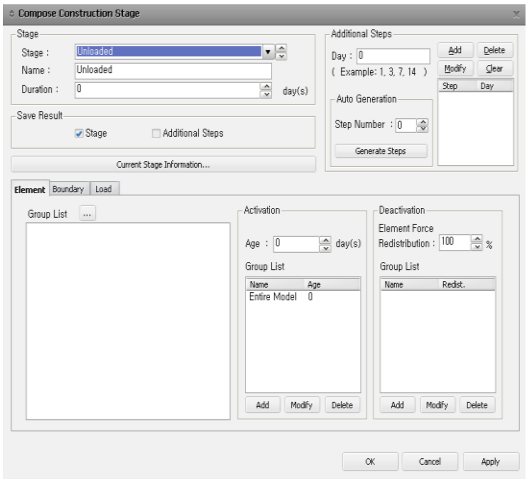
For the staged analysis, *midas CIVIL* Construction Stage function is used. Bi-Linear link for thermal load alone and Bi-Linear link for subsequent loads are defined and applied sequentially.

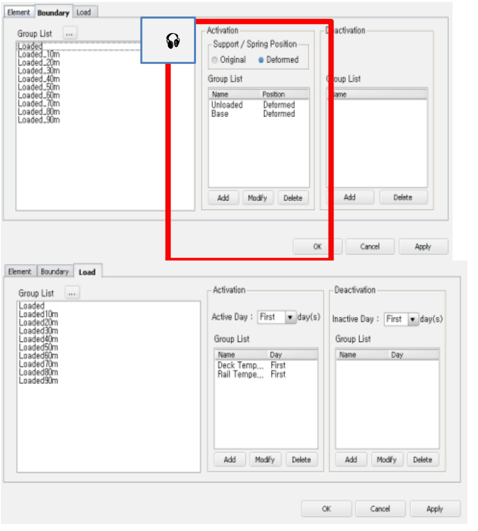
Construction Stage function for the staged analysis is explained in the next page.

3.2 Use *midas CIVIL* Construction Stage Function for the Staged Analysis

Stage	Element	Boundary	Load
Unloaded	Entire Model Active	Base Active Unloaded Active	Deck Temperature Active Rail Temperature Active
Loaded	-	Loaded Active (Deformed) Unloaded Inactive	Loads (Brake Load, Vertical Load) Active

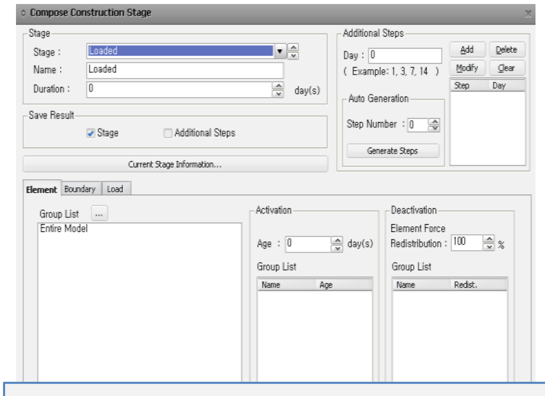
- Define link conditions and load conditions for "Unloaded" stage and "Loaded" stage, using Construction Stage function.
  The boundaries that will be active in "Loaded" stage should be **Deformed Position**.



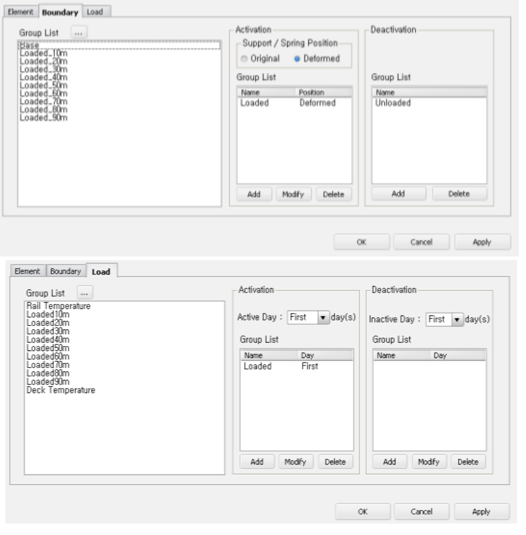


| Construction Stage dialog for the unloaded stage |

- As the train moves, different locations should be analyzed to find the peak compressive stress. The link conditions and the load locations change in the "Loaded" stage. To compare the results, each link condition and load location should be assigned to groups.

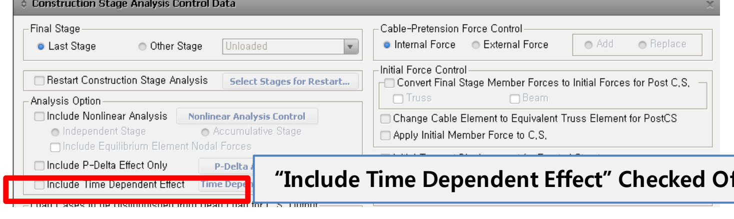


For example, if a load is 10 m distant from the abutment end, Boundary (Loaded_10m) and Load (Loaded10m) should be active.



| Construction Stage dialog for the loaded stage |

- The effects of time dependent materials are not taken into account.



"Include Time Dependent Effect" Checked Off

| Construction Stage Analysis Control Data dialog |

4.1 UIC774-3 Test Case E1-3

4.1.1 Test Model Properties

E1-3 uses Deck Type 1 and the following properties are used for the analysis.

Item	Value	Item	Value
E	2.1×10^5 N/mm^2	Izz	1.02091×10^{-5} m^4
v	0.3	Ixx	4.33934×10^{-6} m^4
α	1.14×10^{-5}	Asy	6.4723×10^{-3} m^2
A	0.0153389 m^2	Asz	1.27397×10^{-2} m^2
Iyy	6.0726×10^{-5} m^4		

| Section Properties of the Rail |

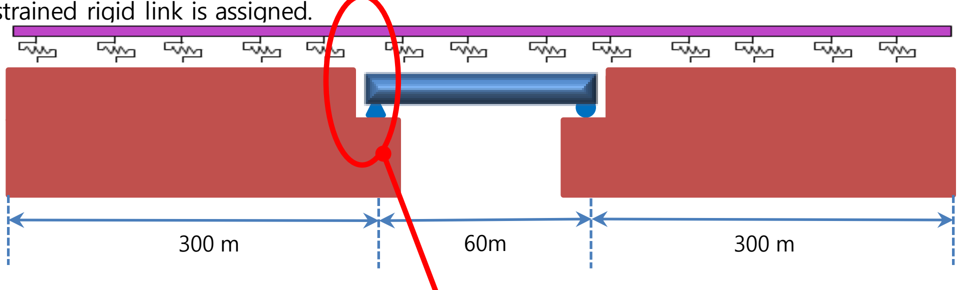
Item	Value	Item	Value
E	2.1×10^5 N/mm^2	Izz	2.59 m^4
v	0.3	Ixx	2.59 m^4
α	1.14×10^{-5}	Asy	= 1000 x A
A	0.74 m^2	Asz	= 1000 x A
Iyy	2.59 m^4	Dsect	6.0 m

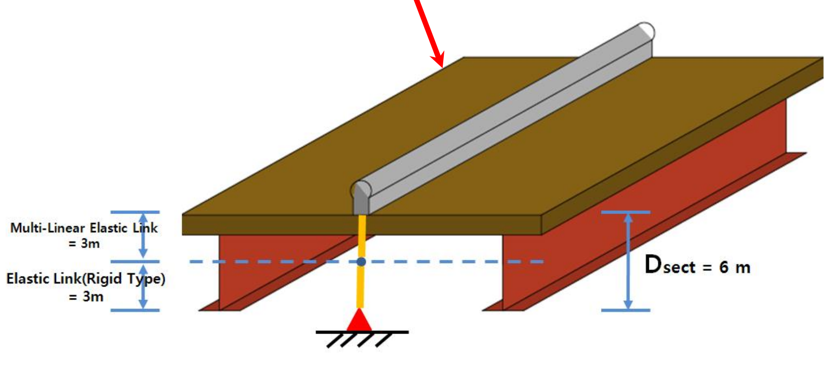
| Section Properties of the Bridge Deck |

4.1.2 Modeling

The element length is 1 m and the total rail length is 660 m including the earthwork and the bridge.

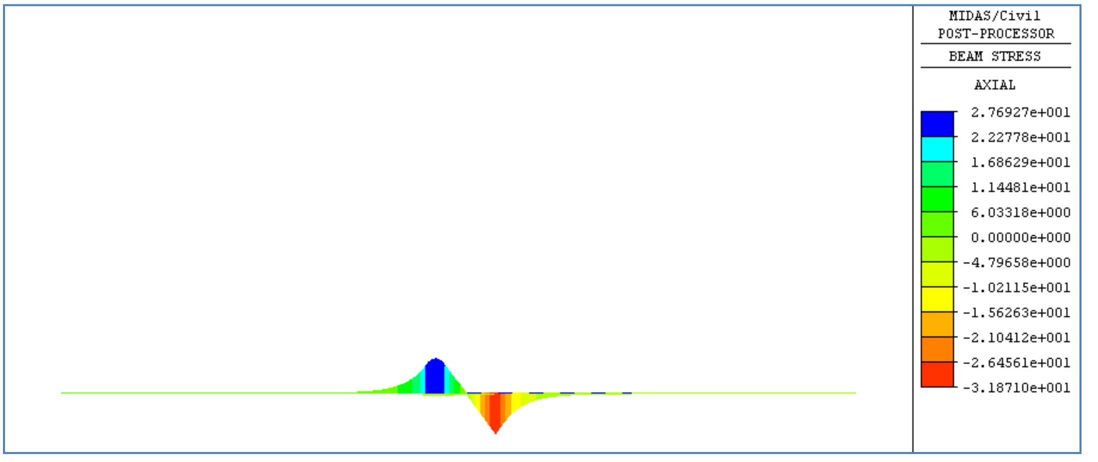
The bridge deck is defined as a Value Type section having the section properties presented in the test case. Because the Value Type section has no section shape, the model is built based on the centroid of the section. At a distance of 3 m (1/2 of the section height) from the centroid of the section, a node is created. Between the node and the centroid, a longitudinally constrained rigid link is assigned.





4.1.3 Apply Temperature Load 35°C to the Deck

Using the "Element Temperature" function, the temperature load to be applied to the bridge deck is defined, and the result is as follows.

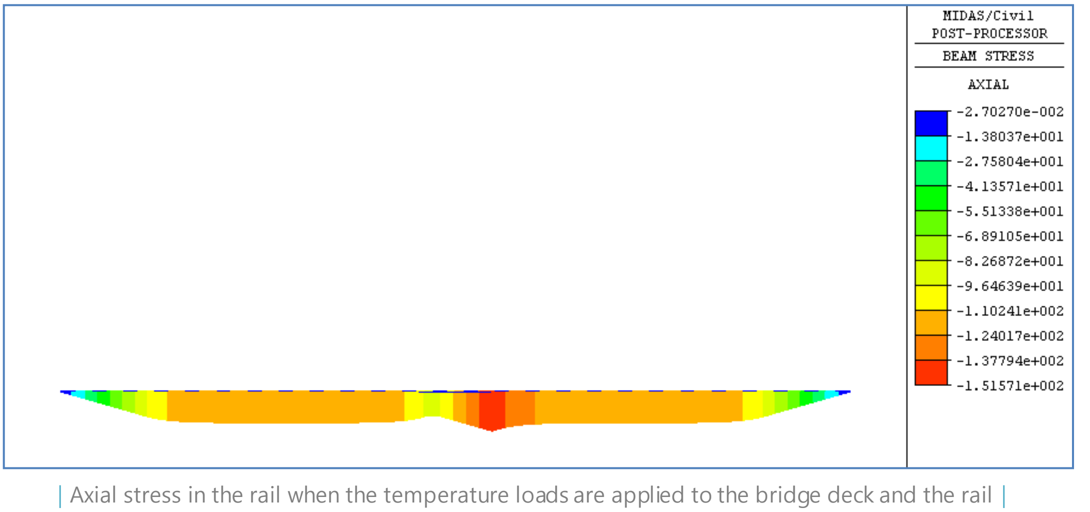


| Axial stress in the rail when the temperature load is applied to the bridge deck|

The axial stress in the rail is 31.87 MPa and matches well the UIC774-3 result 30.67 MPa.

4.1.4 Apply Temperature Load 50°C to the Rail and 35°C to the Deck

Using the "Element Temperature" function, the temperature loads are applied to the rail and the bridge deck, and the result is as follows.

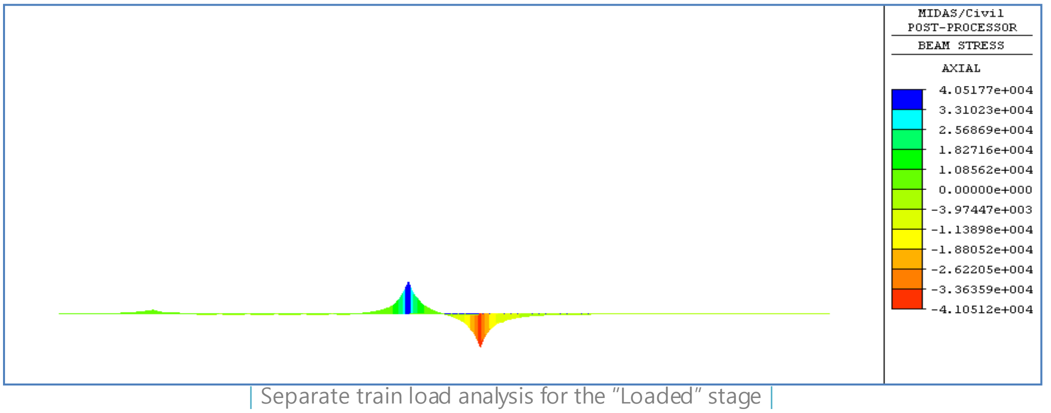


| Axial stress in the rail when the temperature loads are applied to the bridge deck and the rail |

The axial stress in the rail is 151.57 MPa. If the above result (axial stress in the rail due to the temperature load applied to the bridge deck) is excluded, the value comes to 119.7 MPa. This matches well the UIC774-3 result 126 MPa.

4.1.5 Maximum Stress from the Separate Analysis

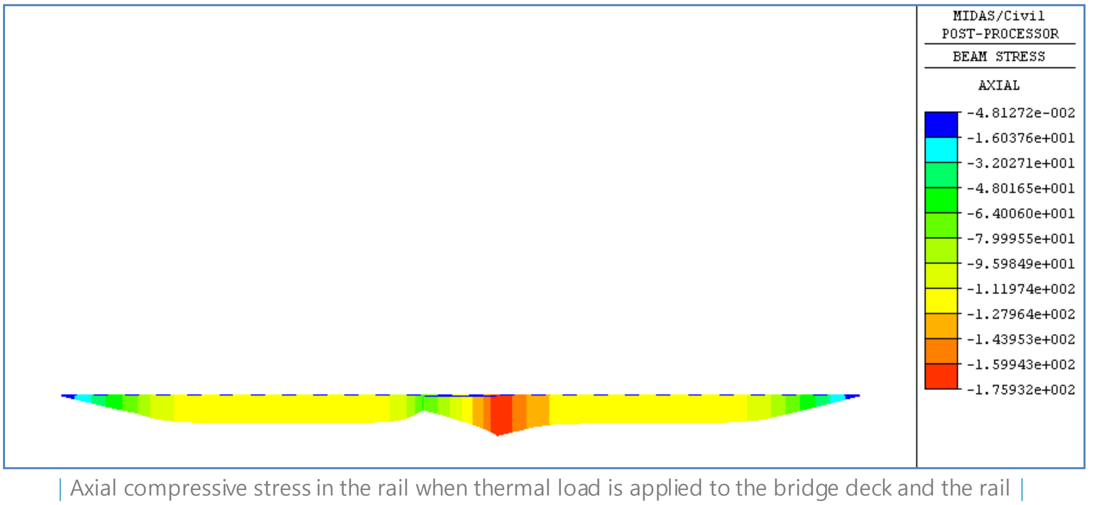
Two models having each load alone are analyzed and the results are combined.
The maximum compressive stress under "Unloaded" stage obtained from the separate analysis is identical to that from the staged analysis. However, the stress under "Loaded" stage from the separate analysis does not include the influence of the temperature. Thus, it assumes that the stress/strain at the initial state of the structure before train loading is applied is zero.
In the "Unloaded" stage it is sufficient to produce one compressive stress due to the temperature load as the boundary condition does not change. However, in the "Loaded" stage, the maximum compressive stress should be determined by moving the train. In the previous page, the compressive stress in the "Unloaded" stage was 151.57 MPa. The maximum compressive stress in the "Loaded" stage is 41.05 MPa as shown in the figure below.
The summation of the compressive stresses due to separate loads is 192.14 MPa, which agrees with 182.4 MPa of UIC774-3 having an error of 5.3%. The maximum error allowed by the Code is 10%.



| Separate train load analysis for the "Loaded" stage |

4.1.6 Apply the Train Load When the Deformation Has Already Occurred Due to the Temperature Load Applied to the Rail and the Bridge Deck

If Boundary Deformed Position of Construction Stage Analysis is applied, it is possible to perform the staged analysis having the "Loaded Stiffness" and maintaining the deformations and internal forces.
The train having the vertical load and the horizontal brake load moves from the left to the right. When the end of the train is located at the end of the bridge deck of the length of 60 m, the rest of the train (240 m) is still loaded on top of the left earthwork. The analysis result is as follows.

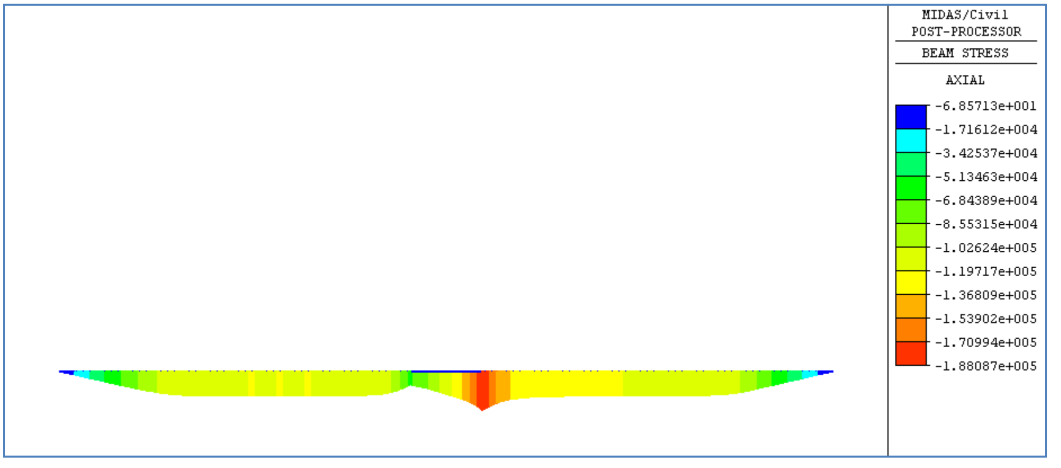


| Axial compressive stress in the rail when thermal load is applied to the bridge deck and the rail |

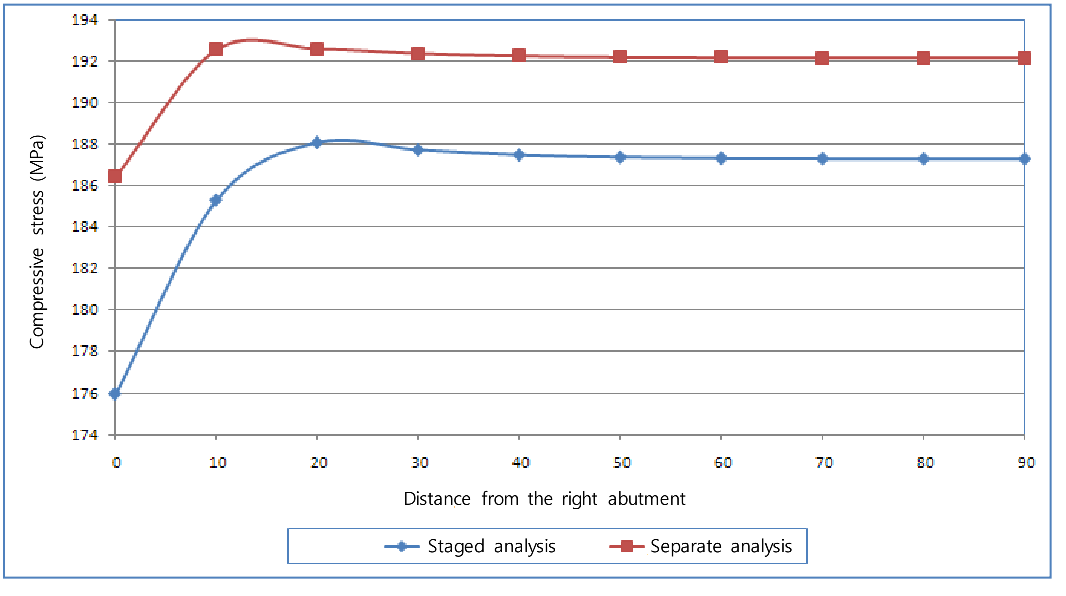
The axial stress in the rail is 175.93 MPa, which agrees with 182.4 MPa of UIC774-3 having an error of 3.5%. The maximum error allowed by the Code is 10%.

4.1.7 Maximum Stress Depending on the Locations of the Train Load

To find the location of the train load for the maximum stress, different train loading locations should be analyzed moving the train from left to right. The train moves from the left abutment to the right by 10 m until it reaches 90 m. The maximum axial stress in the rail obtained from these analyses is 188.09 MPa and the error rate is 2.9%. This matches well as the error is within the maximum error rate 10% allowed by the Code.



| Maximum compressive stress depending on the train load locations |



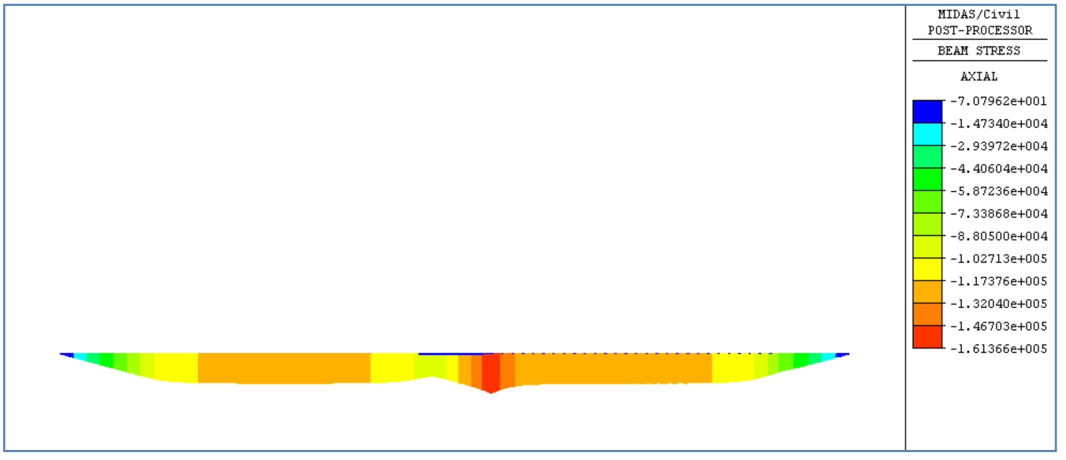
| Graph of the maximum compressive stress depending on the train load locations |

4.2 UIC774-3 Test Case E4-6

The bridge properties for the test case E4-6 are the same as those of the test case E1-3 and the thermal load is also the same. Only the direction of the train load is different.

The result due to the thermal load alone is the same as E1-3, so this result is skipped. Only the result when the train load is applied in addition to the thermal load will be discussed.

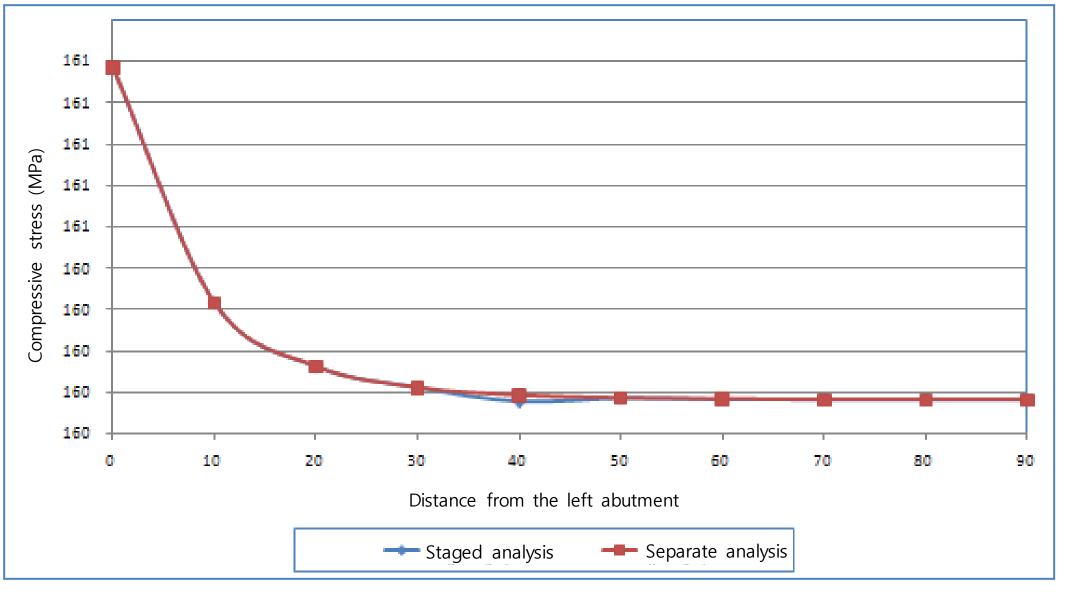
First of all, the train is moved from left to right so that the end of the train is located at the end of the bridge. For this case, the analysis is carried out.



| Axial compressive stress in the Rail when the thermal load and the train load are applied (E4-6) |

The axial stress in the rail is 161.37 MPa, which agrees with 162.06 MPa of UIC774-3 having an error of 0.3% that is below the maximum error 10% allowed by the Code.

To find the location of the train load for the maximum stress, different train loading locations are analyzed moving the train from right to left. The train moves from the right abutment to the left by 10 m until it reaches 90 m. The maximum axial stress in the rail obtained from these analyses is 161.37 MPa and the error rate is 0.3%. In addition, the separate analysis result and the staged analysis result match well.



| Graph of the maximum compressive stress depending on the train load locations |

05. Summary

From the railway bridge analysis using Multi-Linear Elastic Link carried out, we can conclude that the results match well having an error below the maximum error allowed by the Code when compared against the values of UIC774-3.

Whether or not to change the resistance of the ballast depending on the modeling methods shows a slight difference. When a comparison is made between the separate nonlinear analysis of thermal load and train load and the staged analysis using "Deformed Option" of "Construction Stage Analysis" switching the resistance of the ballast from "Unloaded" to "Loaded" under train loading, the results generally match well within the allowable error rate.

In case of test model E1-3, the maximum axial compressive stress occurs when the train load is located at 80 m distant from the left abutment. The staged analysis result is closer to the value of the UIC774-3 Code than the separate analysis result.

In the test model E4-6, even though the simple superposition of nonlinear analyses cannot be accurate, the separate analysis result shows close similarity to the staged analysis result. This is because yield due to thermal load is not observed on the roadbed where train load is applied and therefore, yield due to thermal load and yield due to train load do not overlap.

In conclusion, when separate analysis and staged analysis are carried out for the railway bridge model in accordance with UIC774-3, the axial force can be overestimated in the separate analysis as the separate analysis takes the resistance of the ballast under train loading plus the resistance of the ballast under thermal loading as the resistance of the ballast when train load is applied.

Test Model E1-3

Item	Staged analysis	Separate analysis	UIC Code	Error compared against the UIC Code (Staged / Separate)
Thermal load	-119.70 MPa	-119.70 Mpa	-126.00 MPa	**5.0% / 5.0%**
Max compressive stress	-188.09 MPa	-192.14 Mpa	-182.4 MPa	**2.9% / 5.3%**

Test Model E4-6

Item	Staged analysis	Separate analysis	UIC Code	Error compared against the UIC Code (Staged / Separate)
Thermal load	-119.70 MPa	-119.70 MPa	-126.00 MPa	**5.0% / 5.0%**
Max compressive stress	-161.37 MPa	-161.37 MPa	-162.06 MPa	**0.4% / 0.4%**

| Comparison of the staged/separate analysis results with the UIC Code test results |